



## Normalisation of the Speculative Frame Method and Its Application to the Housing Market in Poland

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**Abstract:** *This article proposes the normalisation of the speculative frame method for identifying real estate bubbles, price shocks, and other disturbances in the real estate market. This index-based method relies on time series data and real estate prices. In this article, the speculative frame method was elaborated and normalised with the use of equations for normalising data sets and research methodologies. The method is discussed on the example of the Polish housing market.*

**Keywords:** speculative frame method; normalisation; bubble; Poland.



## Introduction

Considerable research has been done into price tensions, shocks, and speculative bubbles, which pose significant challenges in the contemporary real estate market. This article attempts to analyse the variations in real estate prices, including price bubbles, and to elaborate on an index-based method for identifying price bubbles and other market disturbances. The main aim of the study was to elaborate on the speculative frame method proposed by Brzezicka and Wisniewski (2021) based on Brzezicka (2017). This index-based method relies on data series relating to real estate prices and transaction dates. The method was normalised by transforming the calculated increase in real estate prices over time to a value approximating one. The article contributes to research into the methodology for detecting real estate prices and diagnosing the real estate market in real time.

Various methods for detecting speculative price bubbles and tensions in the real estate market have been described in the literature. The most popular approaches are based on standard econometric methodologies that aim to determine the differences between the real value of an asset and its fundamental value. In markets characterised by speculative price bubbles, market prices exceed the fundamental value of real estate (Flood and Hodrick 1990: 85). The fundamentals of the real estate market can be analysed from a narrow or a broad perspective. The narrow approach is concerned with the fundamental value, whereas the broader perspective takes into account market variables that can constitute fundamental factors (Hui and Yue 2006: 301; Brzezicka 2020). The narrow approach relies on models of present value, where fundamental value is calculated as the sum of discounted cash flows (Fraser et al. 2008; Mikhed and Zemčík 2009). The broad approach considers the legal, institutional, economic, and demographic factors that influence the performance of the real estate market and are regarded as market fundamentals (Malpezzi 1999; Black et al. 2006; Wang et al., 2018; Mao and Shen 2019). This group of methods includes regression models, cointegration tests, error correction models, and autoregression models (Tomal 2019; Tomal 2020).

Indicator-based procedures and index-based methods can also be used to analyse tensions in the real estate market. In indicator-based procedures, the rate of changes in asset value over time is determined by comparing current indicator values with their long-term average values. These methods are often described by affordability ratios (Himmelberg et al. 2005, Tsai and Peng 2011). The repeat-sales index focuses on different sale prices of the same piece of real estate over time (Clapp and Giaccotto 1998; McMillen and Thorsnes 2006). These methods are widely used on the Polish real estate market. Repeat-sales indices were applied by Czerski et al. (2017) and Głuszak et al. (2018), whereas hedonic indices were developed by Widłak and Tomczyk (2010) and Olszewski et al. (2017). The applicability, methodology, limitations, and advantages of these indices have been discussed by Case et al. (1991), Dorsey et al. (2010), and Hill and Trojanek (2020).

This article examines the house price index methodology and explores the practicality and reliability of such indices. Real time poses a certain challenge in the discussed research methodologies. This problem has been addressed by the rolling window method proposed by Swanson (1998) and the moving window method developed by Phillips et al. (2015a, 2015b). Damianov and Escobari (2016) implemented these concepts in an analysis of the real estate market, but their research involved econometric methods that rely on extensive econometric methods and sets of independent variables to explain the variation in the dependent variable.



Brzezicka and Wisniewski (2021) proposed a method for analysing the real estate market that is based on tensions, shocks, and price bubbles. This approach was referred to as the speculative frame method. In the proposed method, values are computed for a sequence of windows, where the time interval corresponding to the date of the transaction is the fixed horizontal parameter, and the rate of changes in real estate prices is the variable vertical parameter.

The speculative frame method does not rank among the classical econometric methods, and it bears greater resemblance to time series analysis. The explanatory variable is the price-time pair, which is defined as price phenomena that occur in a specific period of time and are inextricably linked with the passage of time. The method takes into account (links between) phenomena that have occurred in the real estate market in the past, which is to say, the accumulated information that is conveyed by market actors.

The basic tenets of the speculative frame method are presented in this paper to promote a better understanding of the described approach. The methodology developed by Brzezicka and Wisniewski is described by equations (1) – (5). In this paper, the speculative method was refined to normalise and determine comparable values with the use of equations (6) – (10) in the Methodology section.

The speculative frame method calculates the mean daily increase ( $DCik_m$ ) in each period of the analysed price and time series, i.e. the mean monthly increase. Time is expressed in days, where 1 January 1900 is denoted as date 1, 2 January 1900 is denoted as date 2, and so on. To calculate a single mean increase in price  $DCik_m$  in period  $m$  with the use of formula (1), the daily increase is determined with the use of formula (2), and it is ordered in in the corresponding time series based on the presented sequence of formulas (3). For the same date, the increase  $DCik$  is divided by 1 rather than by the difference in dates, which resolves the zero increment problem.

$$DCik_m = \frac{\sum_{ik} DCik}{\frac{N!}{2!(N-2)!}} \tag{1}$$

$$DC_{ik} = \frac{x_k - x_i}{t_k - t_i}, \text{ where: } i = 1; \dots; N - 1; k = 2 + (i - 1); \dots; N \tag{2}$$

For  $N = 3$ ,  $DC_{ik}$  is calculated using the following formulas (3):

$$DC_{i=1k=2} = \frac{x_2 - x_1}{t_2 - t_1}, DC_{i=1k=3} = \frac{x_3 - x_1}{t_3 - t_1}, DC_{i=2k=3} = \frac{x_3 - x_2}{t_3 - t_2} \tag{3}$$

In formulas (1) – (3):  $DC_{ik}$  is the increase for  $i$  and  $k$ ;  $n \in \{1, 2, 3, 4, 5, \dots, N\}$ ;  $m \in \{1, 2, 3, 4, 5, \dots, M\}$ ;  $x_1, x_2, \dots, x_N$  – subsequent prices in the analysed period;  $t_1, t_2, \dots, t_N$  – dates of subsequent transactions;  $N$  – number of transactions/last transaction in the adopted time interval;  $M$  – the number of analytical periods and the last analytical period.

The method described by equations (1)-(3) can be used to present price increases in a non-additive manner. In the modified variant of the method, the increase in prices during one period is expressed by a cumulative time-phased increase ( $DCikd_m$ ).



The cumulative increase  $DCikd_m$  expresses the cumulative time-phased increase  $DCik_m$  in period  $m$  ( $m \in \{1, 2, 3, \dots, M\}$ ) according to equation (4). In the first period ( $m = 1$ ), the cumulative increase  $DCikd_1$  is equivalent to the mean increase  $DCik_1$  in the first period. The cumulative increase  $DCikd_m$  sets the conditions for generating an additive model based on the observed increase in prices. In this method, the index  $RDCik_m$  expresses the relationship between the variable vertical dimension of the window, which denotes the rate of increase, and the constant horizontal dimension of the window that denotes time. This relationship is described by equation (5) for  $y < m$ , where  $y$  is the period during which variations in the price increase are observed.

$$DCikd_m = DCik_m + DCikd_{m-1} \text{ for } m > 1; DCikd_m = DCik_1 \text{ for } m = 1 \quad (4)$$

$$RDCikd_m = \frac{DCikd_m - DCikd_{m-y}}{y} \quad (5)$$

The index  $RDCikd_m$  describes the variability in price change, and it supports the determination of the rate of change in real estate prices.

## Methodology

In this article, attempts were made to refine the speculative frame method developed by Brzezicka and Wisniewski (2021) by normalising the results generated by the proposed calculation algorithm. The speculative frame method can be used to calculate the rate of increase in prices, which is expressed by the difference in the vertical dimension of the window and constitutes a basis for analysing the rate of changes in real estate prices in a given period. However, practical analyses have demonstrated that the method should be modified. The results produced by the speculative frame method are not normalised, which implies that they will not be comparable with the results obtained for a different price and time series. This is a common problem when results are not normalised.

The aim of the normalisation procedure was to ensure that the results generated by the speculative frame method are comparable. Normalised values can be used to analyse the relationships between the results obtained for different price and time series. Normalisation supports analyses and comparisons of tensions, shocks, and price bubbles in different real estate markets.

Two additional indices based on the mean increase calculated with formula (1) were introduced to resolve the above problem: the dynamic increase  $DCik1$  and the cumulative dynamic increase  $DCik1d$ . The proposed indices are calculated using equations (5) – (7).

The dynamic increase  $DC1ik$  expresses the relationship between the mean daily increase  $DCik_m$  and the mean price in period  $m$  described by equation (6). The relevant calculations can be based on the arithmetic mean described by equation (7) or the geometric mean given by equation (8). The calculations do not involve a mixed analysis because they are performed separately for the arithmetic mean and the geometric mean.



$$DC1ik_m = \frac{DCik_m}{x_{am}} + 1 \vee DC1ik_m = \frac{DCik_m}{x_{gm}} + 1 \tag{6}$$

$$x_{am} = \frac{x_1 + \dots + x_N}{N} \tag{7}$$

$$x_{gm} = \sqrt[N]{x_1 * x_2 * \dots * x_N} \tag{8}$$

where:  $DCik_m$  – is the mean increase in period  $m$  described by equation (1);  $x_{am}$  – is the arithmetic mean in period  $m$ ;  $x_{gm}$  – is the geometric mean in period  $m$ .

The cumulative dynamic increase ( $DC1ikd_m$ ) expresses the cumulative values of the dynamic increase  $DC1ik_m$ , and it can be used to generate a multiplicative model based on the observed increase in prices using formulas (9) and (10).

$$DC1ikd_m = DC1ik_1 \text{ for } m = 1 \tag{9}$$

$$DC1ikd_m = DC1ik_m * DC1ikd_{m-1} \text{ for } m > 1 \tag{10}$$

where:  $DC1ikd_m$  – is the cumulative dynamic increase in period  $m$ , for  $m \in \{1, 2, 3, \dots, M\}$ ;  $DC1ikd_1$  – is the cumulative dynamic increase in period  $m = 1$ .

In this approach, ‘nominal’ results are transformed into ‘relative’ results that are calibrated to approximate 1. The dynamic increase  $DC1ik_m$  expresses the relationship between the mean daily increase  $DCik_m$  and the mean price in period  $m$  (different in each period). The dynamic increase is divided by the mean, and the result is added to 1 to normalise the results to a value approximating 1. The dynamic increase is a highly accurate normalisation of  $DCik_m$ : the correlation coefficient between  $DCik_m$  and  $DC1ik_m$  ranged from 0.90 to 0.99 in various tests. Like  $DCik_m$ , the values of  $DC1ik_m$  in each period are independent of previous price increments; therefore, they provide information about each individual period. The calculated increase can assume values in the range of  $DC1ik_m \geq 1$  or  $DC1ik_m < 1$ . The applicability and interpretation of  $DC1ik_m$  are based on a local evaluation of results that approximate 1. Results that are close to 1 denote low variations in monthly prices, and results that depart considerably from 1 point to considerable fluctuations in prices. Therefore,  $DC1ik_m$  denotes a local normalised increase in prices in a given period of time.

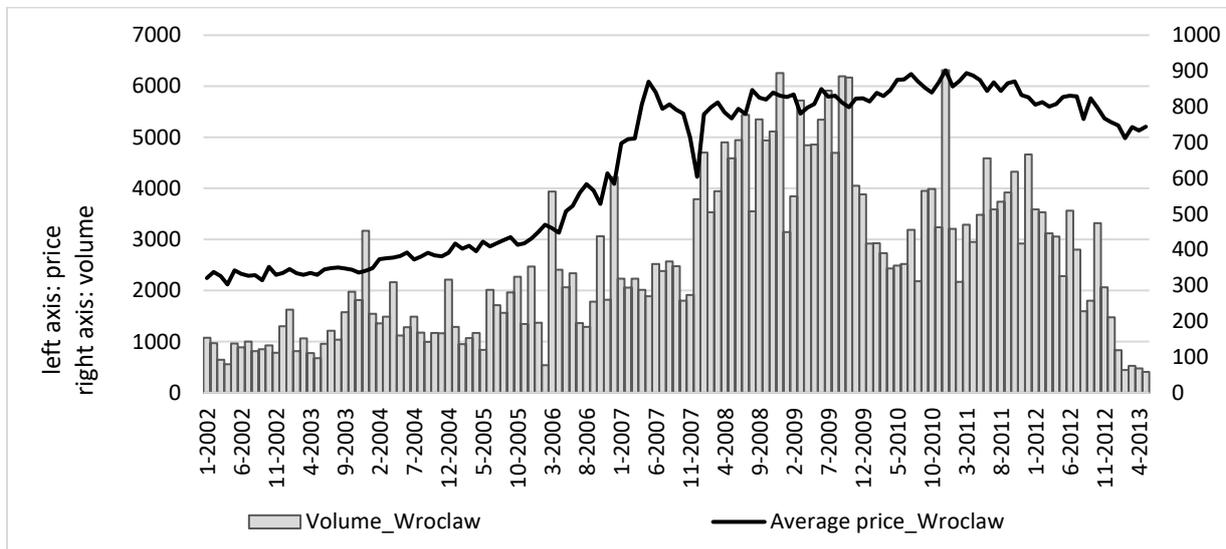
The cumulative dynamic increase  $DC1ikd_m$  expresses the cumulative values of the dynamic increase  $DC1ik_m$ . In each period, the value of  $DC1ikd_m$  is influenced by each preceding increase; therefore, this indicator provides information about each analysed period in view of the results noted in the preceding periods. The values of the dynamic increase  $DC1ik_m$  are aggregated in a multiplicative manner, and since the values of  $DC1ik_m$  are normalised,  $DC1ikd_m$  is also normalised.  $DC1ikd_m$  is the product of previous increments, and it can assume values in the range of  $DC1ikd_m \geq 1$  or  $DC1ikd_m < 1$  after normalisation. If  $DC1ikd_m > 1$ , the product of previous increments was greater than 1. This is possible if the product of the cumulative series from 1 to  $m - 1$  is greater than 1, and  $DC1ikd_m$  in period  $m$  is also greater than 1. Therefore,  $DC1ikd_m$  is the global normalised increase for the entire analysed series.



## Analysis

The proposed elaboration of the speculative frame method was presented on the example of the Polish housing market. A database of real estate prices in Wrocław, the fourth largest Polish city, was analysed in the study. The database was obtained from the AMRON Centre for Analysing and Monitoring the Real Estate Market. The database was released by AMRON solely for research purposes, and it did not contain personal data relating to property sellers or buyers, real estate attributes, or exact locations. Only apartment transactions were analysed. The database contained 50,000 transactions concluded between 2002 and 2013 (1 January 2002 to 31 May 2013). Real estate prices in Wrocław are presented in Figure 1. The left axis presents real estate prices, and the right axis denotes the volume of transactions in the analysed period.

**Figure 1: Real estate prices in Wrocław and the number of concluded transactions**

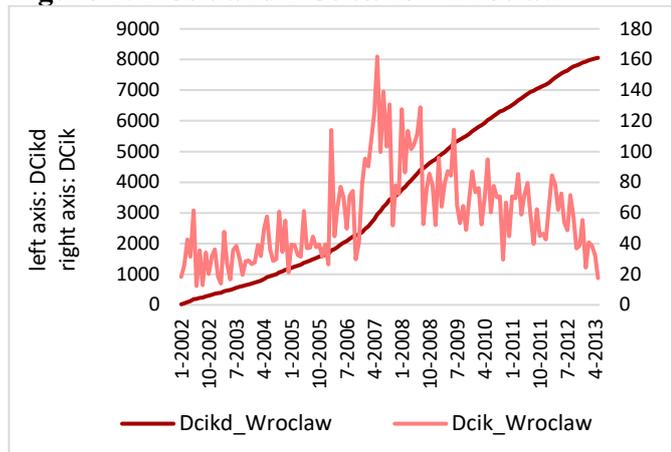


Source: Authors.

A considerable increase in real estate prices was noted in the Polish market in the analysed period. The trends observed in Wrocław did not differ from those reported in other large Polish cities, and the Polish market generally followed European trends. The analysed period was characterised by price increases and adjustments, and it was selected intentionally because market performance during this period was strongly influenced by the global financial crisis of 2008. The calculated values of  $DC_{ik}$  and  $DC_{ikd}$  as well as the values of  $DC_{1ik}$  and  $DC_{1ikd}$  are presented and discussed in this section. All possible relationships between subsequent price increases are presented in Figures 2, 3, 4, and 5. The figures combine two categories of results on different scales; therefore, an additional axis was incorporated in the presentation of results. The axes are labelled in each figure drawing. The normalised increase was calculated based on the arithmetic mean with the use of equations (6) and (7). The geometric mean can also be applied for this purpose in formulas (6) and (8).



Figure 2: *DCik* and *DCikd* for Wrocław



Source: Authors.

Figure 3: *DC1ik* and *DC1ikd* for Wrocław

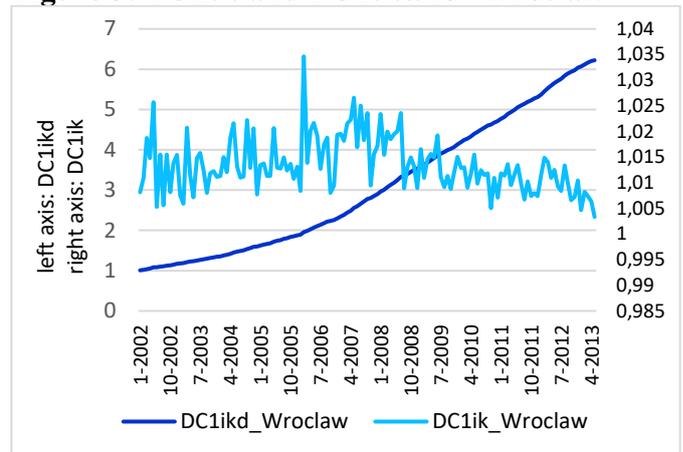
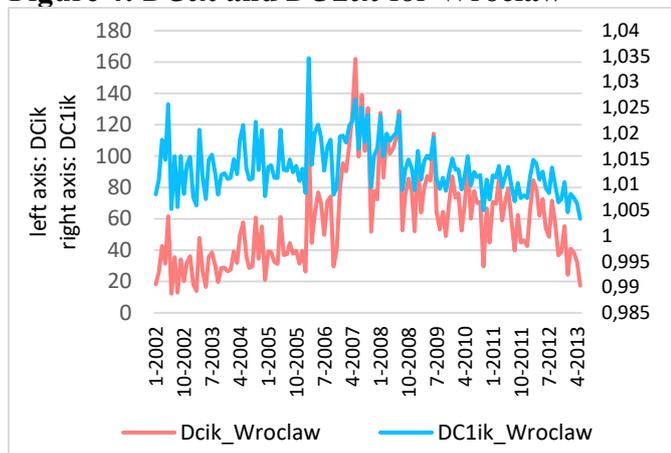
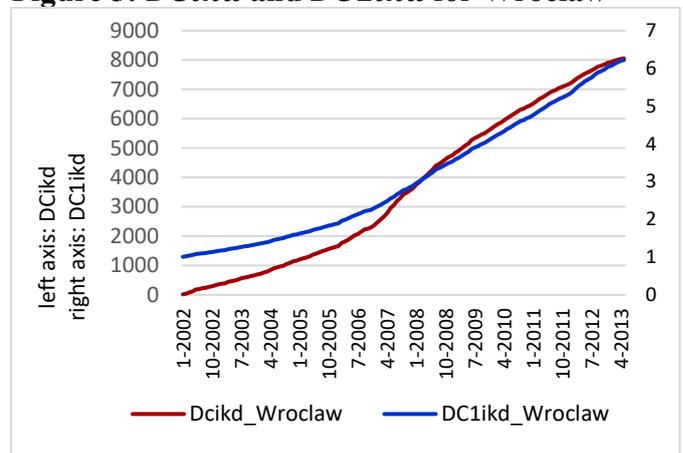


Figure 4: *DCik* and *DC1ik* for Wrocław



Source: Authors.

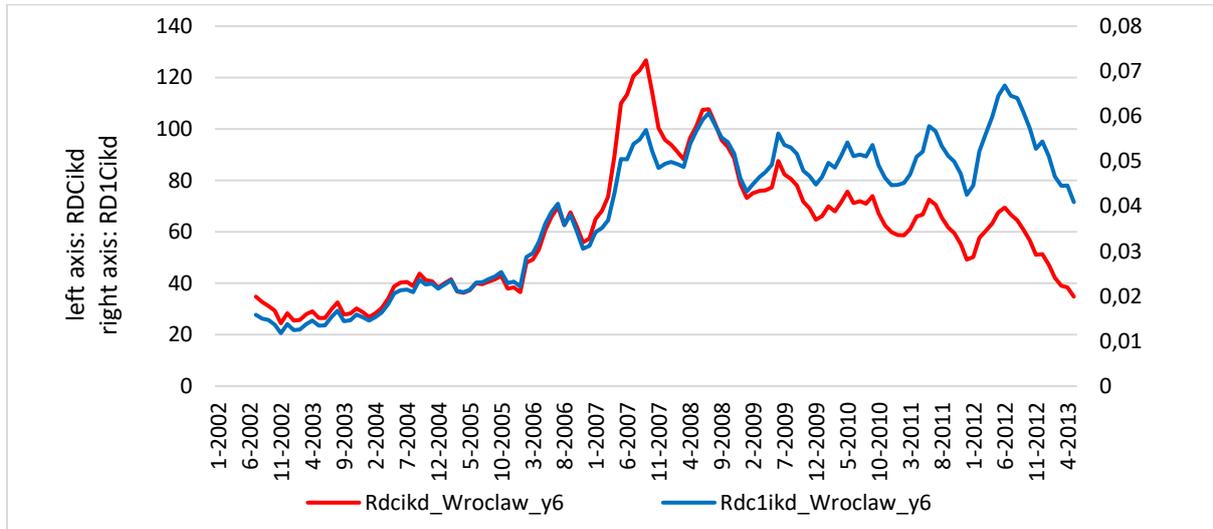
Figure 5: *DCikd* and *DC1ikd* for Wrocław



When the increase in real estate prices in each analysed period is calculated using the normalised method, the original rate of changes is preserved, but the data are adjusted to values approximating 1 (one). This relationship is particularly visible in Figures 3 and 4. Special attention should be paid to the scale of both axes because the point of reference can be preserved in the proposed normalisation method. Original and normalised increments are presented below, but the main aim of the discussed method was to calculate the difference between subsequent increases over time. According to formula (5), the time frame can be 3, 4, or 6 or more months. In the next step, the relative increase was determined based on normalised data. Equation (5) was used for this purpose, but the calculations were performed for both data series: *RDCikd* and *RDC1ikd*. The results are presented in Figure 6. In the discussed method, the additive algorithm for calculating the original increase in prices is transformed into a multiplicative algorithm, and the results are normalised based on *DC1ikd*. In this case, the rate of price changes is also preserved. The normalised method expresses the original increase in prices, but it also accounts for the multiplicative, i.e. absolute, increase in real estate prices. The method is sensitive to price variations in subsequent periods, and it can be used for various analyses of the real estate market.



Figure 6: *RDCikd* and *RDC1ikd* for Wrocław



Source: Authors.

### Conclusion

This article presents a methodology for normalising the results generated by the speculative frame method. A normalisation algorithm was applied to produce results that are comparable and close to 1. This approach enabled a comparison of the results reported on different real estate markets. The normalisation tool has been designed to express the rate of changes in real estate prices. The normalised increase follows the dynamic of the original (non-normalised) increase in prices, and it is equally sensitive to variations in real estate prices. The proposed elaboration of the speculative frame method can be used in various analyses of the real estate market.



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